Definitions

Quantity of Matter Defined

Definition 1

The quantity of matter is the measure of the same arising from its density and magnitude conjointly.

Air of double density, in a space that is also doubled, is quadrupled; in a tripled [space], sextupled. The same is to be understood of snow and powdered substances condensed by compression or liquefaction. And the same account is given of all bodies which are condensed in various ways through various causes. In this I do not take account of the medium (if any) freely pervading the interstices of the parts. Further, in what follows, by the names "body" or "mass" I everywhere mean this quantity. It is apprehended through an individual body's weight. For it is found by experiments with pendulums carried out with the greatest accuracy to be proportional to weight, as will be shown hereafter.

Notes on Definition 1

- **Measure and Proportionality: a Question of Relationships**

"The quantity of matter is the measure of the same arising from its density and magnitude conjointly."

For a present-day student it may not be clear here from Newton's wording that this definition is giving us a proportion, not an equation. When he speaks of the measure "arising," this is one indicator of it being a proportion. The stronger and more explicit indicator is the word "conjointly." Another way to express this would have been that the quantity of matter varies as the density and magnitude conjointly; that is, that two quantities of matter are in the same ratio as the ratio of the densities compounded with the ratio of the volumes.

That Newton is thinking in terms of a proportion is confirmed by the first words in the commentary: "Air of double density, in a space that is also doubled, is quadrupled; in a tripled [space], sextupled."

Newton is not telling us here that we will get a measure of mass (in grams or some such unit) if we multiply a body's density times its bulk. Rather, he is telling us how to compare two or more bodies. The ratio of the masses is the same as the ratio of respective densities compounded with the ratio of their respective bulks or volumes. (Compounding is a way of multiplying

ratios.)

What does Newton mean by "measure"? How does he measure things? In *Principia*, he measures things by looking at relationships, that is, he compares two things of the same sort and finds that they are in the same relationship as the ratio (or compounded ratios) of some other pairs of things, always comparing things of the same sort.

Unlike what we may be accustomed to in physics textbooks, in *Principia* we very seldom get statements that any one thing is *equal to* some particular other thing or combination of other things, for example that a particular mass can be calculated by multiplying a particular density times a particular volume. This language of equations, so familiar to us and so convenient, is almost completely absent from *Principia*.

This is in part because in Newton's time the use of algebra was just starting to establish itself against the traditional use of proportions. But it was also a choice for Newton. He could have used algebra; indeed, elsewhere (and even in a few places in *Principia*) he did use algebra and the analytic calculus he had himself developed. But he evidently felt that what he wanted to represent was better conveyed by the language of proportions.

Though cumbersome, the insistence on sticking with proportions did two things that are lost in converting to algebra and equations (where that conversion would have been possible).

First, it stayed clear about the meanings of the quantities being worked with. For example, we understand that one distance might be twice another distance, but whatever could be the meaning of dividing a distance by a force?

Second, it kept things in terms of relationships. One might want to speculate about why that might have been a value to Newton, and to watch as one works through *Principia* for other ways in that Newton seems to be seeing relationships as the way to understand foundational things. Keep this in mind for pondering the meaning of the Third Law of Motion and the understanding of the working of gravity itself.

- **Nature of a Definition:** This section calls itself "Definitions." How are we to understand what is said about things described in a definition, as opposed, say, to what is said in something called a law?

This may not be obvious. For example, a student once asked, "Why does Newton define quantity of matter in terms of density when it seems that we know how to measure quantity of matter directly, but we don't know how to measure density directly?"

This question raises several interesting issues, two of which may be helpful to explore here.

First, the question reveals a confusion between a definition and a procedure for measuring the defined item in a laboratory. Definitions in a mathematical exposition describe how terms will be used and do not establish that the thing defined exists. And if the thing defined does exist,

the definition doesn't provide the construction or method of finding it. That may need to come in a later proposition, or perhaps in a postulate. How one would go about assigning a number to the quantity of matter in a given thing in the world is not the concern in the definition (or generally in *Principia*). Rather, he is laying out here the relationship between quantity of matter, density, and size.

Another sort of difficulty with this question is that it claims that we know how to measure quantity of matter directly. Do we? We have balance scales that compare weights and spring scales that measure the force we call weight. But weight and quantity of matter, force and stuff, are different things, and we don't yet know the relationship between them. That's one of the important things worked out in *Principia* (see the next note).

This first definition that opens *Principia* invites us to start thinking about just what mass (or quantity of matter) could be on its own. **Question for Discussion.** Imagine that we have no Newtonian physics, and we're trying to sort out these things we're going to be working with. What might quantity of matter be? How would we want to define it, how would we give people a picture of what we will mean when we use the words? Then consider Newton's answers to those questions in this definition.

We have a tendency, living in a post-Newtonian world, to think of mass as something absolute, the concept created along with the matter itself, and that anyone with any other concept or definition must be ignorant or confused. But many ways were open to Newton in conceptualizing what we now call mass. The way he did decide to think about it, the way he defined it, turned out to be very useful, so useful that it's now hard to imagine any other way of thinking about it.

The way to think about these definitions is not to ask whether we would have picked different words (or equations) to describe what we as his beneficiaries understand these things to be. That might be appropriate if he had been given the understanding modern physics uses and was just writing a textbook to codify it. Then we (or some other modern textbook we have read) might have different or better ways of expressing this understanding.

But our opportunity here is to do something much more thrilling and engaging than fine-tuning a textbook. Rather, we can step back and stand with Newton at the moment of formulating what a useful way of thinking about these things would be.

- **Quantity of Matter and Weight**

"[Mass] is apprehended through an individual body's weight. For it is found by experiments with pendulums carried out with the greatest accuracy to be proportional to weight, as will be shown hereafter."

As Newton will be doing frequently in his commentaries to the definitions and the laws of motion, he is here looking ahead to what will emerge from this work as a whole, and in particular to what will prove to be true when

we pull in information about how things work in our world. In this case, as we embark on this investigation, we don't know the relationship between mass and weight, and these experiments he is referring to are part of the development that *Principia* will lead us through. That mass, or quantity of matter, can be "apprehended through weight" must be shown, and it is indeed finally shown, but not until Book III, which makes the various applications to our actual world. The experiments Newton mentions here are cited, and the relationship of mass and weight established, in Book III Proposition 6, which is not included in this selection.

- **Mass and Quantity of Matter:** The word here translated as "mass" is Latin "*massa.*" *Massa* is a large irregular lump of something, a bundle or heap. This is in contrast to our modern use of the word mass, which is more technical and abstract. The technical and abstract term for Newton, corresponding to our modern use of the word mass, was "quantity of matter."

The translation in this selection always distinguishes *massa,* mass, from *quantitas materiae,* quantity of matter.

You won't get into any trouble in reading *Principia* if you mentally substitute "mass" for "quantity of matter." But you might want to notice when Newton uses each term, keeping in mind that, when he used the word mass as opposed to the term quantity of matter, he was likely picturing something in its individuality as a lump of matter.

Quantity of Motion Defined

Definition 2

The quantity of motion is the measure of the same arising from the velocity and the quantity of matter conjointly.

The motion of the whole is the sum of the motions in the individual parts, and therefore in a body twice as big, with an equal velocity, it is doubled, and with a doubled velocity it is quadrupled.

Note on Definition 2

This definition is expressed as a proportion, not as an equation. See the first note to Definition 1.

Again, we are invited to think about what quantity of motion might be. **Question for Discussion.** What is motion itself? How might one measure it (not necessarily in the laboratory, but in thought)? One might have chosen

to think of motion as the movement itself, maybe measured by the distance traveled in a particular time, or even just the total distance traveled. (In casual speech, we do sometimes use the word in both these ways.)

Newton chooses in *Principia* to use a definition of motion as the speed and the quantity of matter conjointly—that is, the ratio of their corresponding speeds compounded with the ratio of their corresponding quantities of matter. **Question for Discussion.** What does this tell us about the way he is thinking of motion? How is his picture different from other definitions someone might have made?

Kinds of Force (Definitions 3–8)

Newton defines two types of force for us in this section. The first is *vis insita,* or inherent force, given in Definition 3. The second is *vis impressa,* impressed force, given in Definition 4. In Definition 5 he goes on to describe a particular type of impressed force, "centripetal force," the investigation of which is central to this book. He then further distinguishes three different measures of this sort of impressed force, the absolute, the accelerative, and the motive quantities of centripetal force, in Definitions 6–8 and the following commentary.

Inherent Force (Inertia, *vis insita*) Defined

Definition 3

The inherent force of matter is the power of resisting, by which each and every body, to the extent that it can, perseveres in its state either of resting or of moving uniformly in a straight line.

This is always proportional to its body, and does not differ in any way from the inertia of mass, except in the mode of conception. Through the inertia of matter it comes to be that every body is with difficulty disturbed from its state either of resting or of moving. Whence the inherent force can also be called by the extremely significant name, "force of Inertia." A body exercises this force only in the alteration of its status by another force being impressed upon it, and this exercise falls under the diverse considerations of resistance and impetus: resistance, to the extent that a body resists an impressed force in order to preserve its state, and impetus, to the extent that the same body, in giving way with difficulty to the force of a resisting obstacle, endeavors to change the state of that

obstacle. Common opinion attributes resistance to things at rest and impetus to things in motion, but motion and rest, as they are commonly conceived, are distinguished from each other only with respect [to each other], nor are those things really at rest which are commonly seen as if at rest.

Notes on Definition 3

- This is the definition of inherent force, saying, in the way of definitions, how the term will be used. The existence of inherent force is postulated in Law 1 where it is asserted that every body continues in its state of motion or rest unless driven to alter that state by an impressed force. Definition 3 and Law 1 are best read in conjunction by anyone attempting to understand this phenomenon.

- The word inertia was not a familiar physical term when Newton used it in this book. It had first been used in a physical sense by Kepler, for whom it meant a body's tendency to come to rest. Newton borrowed the word, but changed its meaning to fit his physics. The basic meaning is "laziness" or "sluggishness."

- We must be especially careful not to project modern textbook definitions or concepts of inertia and force onto Newton's definitions. We're in another world as we read *Principia,* a world we have entered by time travel. Only careful alert exploration will reveal how much of what we find here is what we're accustomed to at home and how much will be different in interesting and even instructive (but possibly subtle) ways. Concepts evolve over time. Words change their meaning. Formulas of modern physics that claim to be "Newtonian" may in fact use different definitions of the same terms. There is no one eternal meaning of such terms as force. Rather, these are concepts that evolve over time. As we read Newton's words, we're at one snapshot of time—a very important moment, but one more than three centuries in the past.

Question for Discussion. What is our own understanding of force? I don't mean to ask what formula we learned in high school physics. What is this thing in the world, in our experience, that we call force? We can ask how we use the word when we're talking about our own personal experience of the world, when describing our everyday experience with material objects and when using the world analogously for psychological, social, or political phenomena. Do we have a direct experience of this thing (or maybe a set of things having something in common) that leads us to a concept prior to any definitions or formulas we might learn in school?

- Returning now to Newton, and looking at the wording of this definition, we might wonder what sort of force this could be. He says that it is that

something that is proportional only to body (note that he says in Definition 1 that when he uses the terms body and mass he means quantity of matter). If inherent force is proportional "only" to quantity of matter it is not, presumably, proportional to anything else (and he doesn't mention anything else). But how is it that something proportional only to body comes to act as a force, and not only act in some force-like way, but be *defined* as force?

One notes that not only does he define this using the word force (*vis*), but he goes out of his way to emphasize that it is important to him to so classify it, calling it an "extremely significant" name:

"Through the inertia of matter it comes to be that every body is with difficulty disturbed from its state either of resting or of moving. Whence the inherent force can also be called by the extremely significant name, 'force of Inertia.'"

Question for Discussion. It is worth speculating before we move on about the way Newton is understanding this force of inertia or inherent force, considering in what sense he is seeing it as a force (*vis*), with the possible insights such thinking might yield about what force means for him. A look ahead at the impressed force of Definition 4 might give something to go on. Is there a way in which inherent force can be seen acting in a similar way to the way he says impressed force acts? How is it different? He calls *vis insita* a "power of resisting." How would that be measured?

- If it seems that the more carefully you read and question the text the more puzzles you find, don't feel discouraged or yield to a temptation to fall back on modern formulas or understandings. You don't expect to have figured out "whodunit" in the first pages of a mystery novel. Rather, let that experience of puzzlement awaken a sense of wonder and give you something to ponder as you go along through *Principia*. At this early point we really don't have a great deal to go on. We haven't seen how he will be speaking about force in propositions or even in the Laws of Motion. We will need to stay alert for how he applies these terms in what follows and continue to feel out his vision and intent.

- "...motion and rest, as they are commonly conceived, are distinguished from each other only with respect [to each other], nor are those things really at rest which are commonly seen as if at rest."

Notice Newton's assertion here of the relativity of motion and rest, "as they are commonly conceived." That is, from our vantage point, we can't tell whether things are absolutely moving or at rest: we can only say whether they are moving or at rest with respect ourselves or with respect to one other. He will take up this question of relativity of motion and whether there is such a thing as knowable absolute motion in the scholium following these definitions.