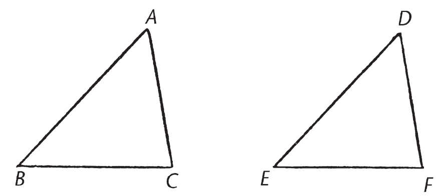
Proposition 4

If two triangles have the two sides equal to two sides respectively, and have the angles contained by the equal straight lines equal, they will also have the base equal to the base, the triangle will be equal to the triangle, and the remaining angles will be equal to the remaining angles respectively, namely those which the equal sides subtend.

Let *ABC, DEF* be two triangles having the two sides *AB, AC* equal to the two sides *DE, DF* respectively, namely *AB* to *DE* and *AC* to *DF,* and the angle *BAC* equal to the angle *EDF.*



I say that the base *BC* is also equal to the base *EF,* the triangle *ABC* will be equal to the triangle *DEF,* and the remaining angles will be equal to the remaining angles respectively, namely those which the equal sides subtend, that is, the angle *ABC* to the angle *DEF,* and the angle *ACB* to the angle *DFE.*

For, if the triangle *ABC* be applied to the triangle *DEF,* and if the point *A* be placed on the point *D* and the straight line *AB* on *DE,* then the point *B* will also coincide with *E,* because *AB* is equal to *DE.*

Again, *AB* coinciding with *DE,* the straight line *AC* will also coincide with *DF,* because the angle *BAC* is equal to the angle *EDF*;
hence the point *C* will also coincide with the point *F,* because *AC* is again equal to *DF.*

But *B* also coincided with *E*;
hence the base *BC* will coincide with the base *EF,*
and will be equal to it. [C.N. 4]

Thus the whole triangle *ABC* will coincide with the whole triangle *DEF,*
and will be equal to it. [C.N. 4]

And the remaining angles will also coincide with the remaining angles and will be equal to them,
the angle *ABC* to the angle *DEF,*
and the angle *ACB* to the angle *DFE.* [C.N. 4]

Therefore etc.*

Q.E.D.

* "Therefore, etc." and the abbreviations Q.E.D. and Q.E.F. are explained on pages xi and xii. —Ed.

Sample Questions: Proposition 4

- The first three propositions were things to do. Now we shift and get the first of many propositions that demonstrate something. How do you see the difference in approach, strategy, or language?
- This is the first time he says "I say that…" after stating the enunciation. Why does he precede his demonstration with these words? Is it that in the previous propositions he is declaring a truth independent of himself: do this and you will have that, but in this and subsequent propositions, he is taking responsibility for the statement in the enunciation and proposing to make an argument for it? Saying that you don't have to believe him until you see at the end of the demonstration whether you are persuaded?
- Here is our first appearance of an assertion of equality of areas ("the triangle will be equal to the triangle" and we know the figure is that which is contained by the boundary). Through this book Euclid will build more complex conclusions about equality of areas.
- Here our foundation is laid on coincidence, based on Common Notion 4 that things which coincide are equal. At this point in building up our system, we have been using the simply arithmetical axioms of the first three common notions and now the somewhat different criterion of coincidence from Common Notion 4. Later we will also call upon the defined (Def. 10) and postulated (Post. 4) equality of right angles.
- This is also the first of the congruence proofs. Can you think of any way to prove that the areas, the remaining angles, and the base are equal without using coincidence?
- How do we know that these triangles, and their points, lines, and angles can be made to coincide? We can't pick up one triangle and move it onto the other. What do you think about his argument, his moving-in-thought by first placing a point to coincide, then the lines around the angle at that point? Does doing this step by step get around the strangeness of imagining that we can pick up and move the triangle without its being changed somehow?
- And if the conclusion follows from Common Notion 4, why do we need this proposition? Because we have to *show* that they coincide? If so, we need to look carefully at the wording: what's given and what's proved at each step.
- Do these steps themselves need authorization? What about when he says that *B* will coincide with *E* because they are equal in length? Those steps do seem able to be immediately inferred; in fact it is hard to picture the lines *not* coinciding. It they didn't coincide the two straight lines would enclose a space. Can you deduce their coincidence logically by putting together some combination of previous elements, such as Def. 4, Postulates 1 and 2, and Common Notion 4? Or is something still missing?

[CONTINUED ▸]

Sample Questions for Proposition 4, continued:

- It has been suggested that Euclid should have just made I.4 a postulate. Why do you think he made it a theorem instead?
- If it is legitimate to pick something up and move it, why couldn't we pick up and move the lines in the first three propositions? Is there something different in those cases? Or is it just that Euclid is not comfortable with this method and does it only when there is no alternative? (That turns out to be only two places: I.4 and I.8.)
- So can we be sure that the triangle was not changed by being moved? And once we have established that the two *superimposed* triangles are equal, how can this be extended to those that are *not* superimposed?